Pressure Vessel Strength Analysis as Conjugation of Cylindrical and Spherical Shells

Ju. S. Stepanov V. A. Gordon

(Oryol State Technical University, Russia)

Abstract The present investigation is dedicated to the analysis of the strainly-deformed state of the shell conjugation as a spherical band with two cylindrical shells, loaded with the uniform one simulates part of the hydraulic pump casing. The investigations method is analytical, the area of use the results gained-design computations.

Keywords pump, shell, conjugation, tension, deformation, strength

This paper is the first amongst those, prepared by the authors for publishing and dedicated to the analysis of the strainly-deformed state of power parts in pressurized electropumps: casing, flanges, stator sleeves etc, dimensioning of the fixtures continuous strength.

The aim of the explortions performed consists in the elaboration of the analytical methods of testing the power parts strength of pressurized pumps, the results of which can serve as the argument and basis for a number of design solutions, aimed to the reduction of the material consumption and production costs.

1 Model description

A single-stage pressurized pump represents a complex axis-symmetrical structure, where the computation model of the power unit of which can serve the conjugation of casing Ⅰ -of the cylindrical shell, of the front Ⅱ and the real Ⅲ bottoms in the form of spherical bands and of the suction Ⅳ and pressure Ⅴ branch connection pipes with flanges(Fig. 1). As a computation loading we assume the permanent operative pressure p.

The analysis of the known results and experimental data show, that the maximum equivalent tensions increase in the spherical area Ⅱ of the assembly.

Now consider the jointing area of the cylindrical shell Ⅰ (radius R_1 and wall thickness δ_1) and the spherical bottom Ⅱ (radius R_2 and wall thickness δ_2). To the conjugation point of the spherical area Ⅱ with the cylindrical one Ⅰ corresponds the angle θ_0 ($\sin\theta_0 = \frac{R_1}{R_2}$), and to the conjugation point of spherical area Ⅱ with the suction branch pipe Ⅳ corresponds the angle θ_* ($\sin\theta_* = \frac{R_3}{R_2}$) (Fig.2).

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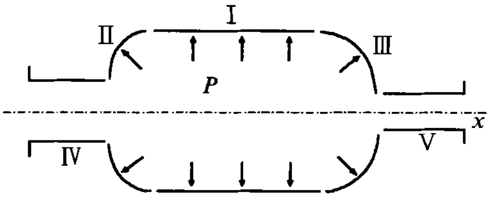

Fig.1 The computation model

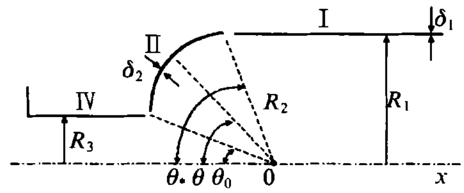

Fig.2 The conjugation part

We′ ll introduce designations: index 《1》 above means, that the corresponding value regards the cylindrical one, index 《0》 below relates to the values of the momentless tension state, and index 《k》 regards the values of the edge effect. We′ ll consider the branch pipe Ⅳ to be absolutely rigid.

Mentally we′ ll divide both shells and their effect upon each other replace with in known forces and moments on the edges(Fig. 3).

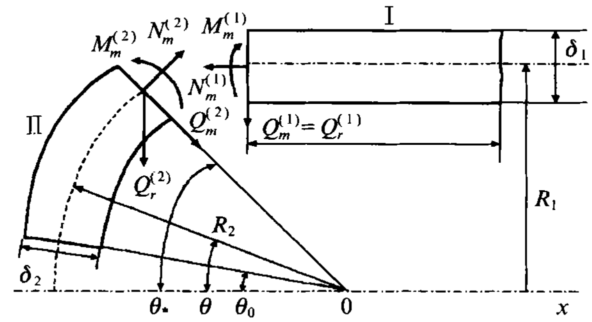

Fig.3 Sizes and internal force

And all the values of the moment tension state in the moment tension state in the joint area of shell we′ ll express through w_k—— displacement in the direction of the normale to the middle surface($w_0^{(1)} = -u_{r0}^{(1)}$), haning put them down in the form

$$w_k(s) = e^{-\beta s}(C_1\cos\beta s + C_2\sin\beta s) \tag{1}$$

wheres: s——are coordinate ($s = R_2\theta$); β——attenuation coefficient.

The function (2, 3) is the solution for the edge effect of the equation of axissymmetrical shell bending

$$\frac{d^4 w_k}{d s^4} + 4\beta^4 w_k = 0$$

For the cylindrical shell

$$w_k^{(1)} = e^{-\beta_1 x}(\Delta_1\cos\beta_1 x + B_1\sin\beta_1 x) \tag{2}$$

For the spherical shell

$$w_k^{(2)} = e^{-\beta_2(\theta_0-\theta)}[A_2\cos\beta_2(\theta_0-\theta) + B_2\sin\beta_2(\theta_0-\theta)] \tag{3}$$

where $\beta_1 = \sqrt[4]{\frac{3(1-\mu^2)}{(R_1\delta_1)^2}}$, $\beta_2 = \sqrt{3(1-\mu^2)(\frac{\delta_2}{R_2})^2}$

u——, Poisson′ s ratio for the shell material.

The moments and cross-cutting forces one determines in the following way[2]

$$M_m = -D\frac{d^2 w_k}{d s^2} = 2D\beta^2 e^{-\beta s}(C_2\cos\beta s - C_1\sin\beta s)$$

$$M_t = \mu M_m$$

$$Q_m = -D\frac{d^3 w_k}{d s^3} = -2D\beta^3 e^{-\beta s}[C_1(\cos\beta s - \sin\beta s) + C_2(\sin\beta s + \cos\beta s)] \tag{4}$$

$$N_{mk} = Q_m\cot\theta$$

$$N_{tk} = -\frac{E\delta}{R_2}e^{-\beta s}(C_1\cos\beta s + C_2\sin\beta s) \tag{5}$$

where: M_m, M_t——meridional and tangentional bending moments; Q_m——cross-cutting force; N_m——

diaphragm force; $D = \dfrac{E\delta^3}{12(1-\mu^2)}$——linear cylindrical shell rigidity; E——Youmg's modulus.

Instead of force N_m an Q_m on the shell edge it is more convenient to consider the vertical Q_η and the horizontal Q_r components, determined with following relations

$$Q_\eta = N_m \sin\theta - Q_m \cos\theta = N_{m0}\sin\theta + N_{mk}\sin\theta - Q_m\cos\theta$$
$$Q_r = N_m\cos\theta + Q_m\sin\theta = N_{m0}\cos\theta + N_{mk}\cos\theta + Q_m\sin\theta$$

where from, taking into account (5), we'll receive

$$Q_r = N_{m0}\cos\theta + \frac{Q_m}{\sin\theta},\quad Q_\eta = N_{m0}\sin\theta \tag{6}$$

The rotation angle of the meridian α displacement components u_r and u_η one determines in the following way

$$\alpha = \frac{u}{R_m} + \frac{1}{R_m}\frac{dw}{d\theta} = \frac{1}{R_m}\left(u_0 + \frac{dw_0}{d\theta}\right) - \frac{dw_k}{ds} \tag{7}$$

where $ds = -R_m d\theta$, R_m—— radius of the meridian arc.

The radial displacement of the parallel circle radius

$$u_r = u_0\cos\theta - w_0\sin\theta - w_k\sin\theta \tag{8}$$

and axial displacement

$$u_\eta = u_0\sin\theta + w_0\cos\theta + w_k\cos\theta$$

The kinematic conditions of the conjugation have a view

$$\sum_{i=1}^{2} u_r^{(i)} = 0,\quad \sum_{i=1}^{2}\alpha^{(i)} = 0$$

where $u_r^{(i)}$—— radial displacement ($i = 1, 2$); $\alpha^{(i)}$—— rotation angle of both conjugated shells, that is

$$\begin{aligned} u_{r0}^{(1)} + u_{rk}^{(1)} &= u_{r0}^{(2)} + u_{rk}^{(2)} \\ \alpha_k^{(1)} &= -\alpha_k^{(2)} \end{aligned} \tag{9}$$

Static conditions of the conjugation for that case have a view

$$M_m^{(1)} = M_m^{(2)};\quad Q_{rk}^{(1)} = -Q_{r0}^{(2)} - Q_{rk}^{(2)} \tag{10}$$

In the assumed designations the momentless solution on the edge at $x = 0$ will have a view[2]

$$\begin{aligned} u_{r0}^{(1)} &= -\frac{2-\mu}{2E\delta_1}pR_1^2;\quad \alpha_0^{(1)} = 0 \\ M_{m0}^{(1)} &= 0;\quad Q_{r0}^{(1)} = 0 \end{aligned} \tag{11}$$

and for spherical shell at $\theta = \theta_0$

$$\begin{aligned} u_{r0}^{(2)} &= -\frac{1-\mu}{2E\delta_2}pR_2\sin\theta_0;\quad \alpha_0^{(2)} = 0 \\ M_{m0}^{(2)} &= 0;\quad Q_{r0}^{(2)} = -\frac{1}{2}pR_2\cos\theta_0;\quad N_{m0} = \frac{1}{2}pR_2 \end{aligned} \tag{12}$$

Using the relations (8)、(10)~(12) for cylindrical shell in the joint cross-section at $x = 0$ we'll have

$$\begin{aligned} u_{rk}^{(1)} &= (w_k^{(1)})_{x=0} = A_1;\quad \alpha_k^{(1)} = \left(\frac{d\,w_k^{(1)}}{dx}\right)_{x=0} = -\beta_1(A_1 - B_1) \\ M_{mk}^{(1)} &= D_1\left(\frac{d^2 w_k^{(1)}}{dx^2}\right)_{x=0} = -2D_1\beta_1^2 B_1 \\ Q_{rk}^{(1)} &= Q_{mk}^{(1)} = D_1\left(\frac{d^3 w_k^{(1)}}{dx^3}\right)_{x=0} = 2D_1\beta_1^3(A_1 + B_1) \end{aligned} \tag{13}$$

and for spherical shell at $\theta = \theta_0$

$$
\begin{aligned}
u_{rk}^{(2)} &= (w_k^{(2)} \sin\theta)_{\theta=\theta_0} = A_2 \sin\theta_0 \\
\alpha_k^{(2)} &= \left(\frac{\mathrm{d}w_k^{(2)}}{R_2 \mathrm{d}\theta}\right)_{\theta=\theta_0} = -\frac{\beta_2}{R_2}(A_2 - B_2) \\
M_{mk}^{(2)} &= D_2 \left(\frac{\mathrm{d}^2 w_k^{(2)}}{R_2^2 \mathrm{d}\theta^2}\right)_{\theta=\theta_0} = -2D_2 \frac{\beta_2^2}{R_2^2} B_2 \\
Q_{rk}^{(2)} &= \frac{D_2}{\sin\theta_0}\left(\frac{\mathrm{d}^3 w_k^{(2)}}{R_2^3 \mathrm{d}\theta^3}\right)_{\theta=\theta_0} = \frac{2D_2 \beta_2^3}{R_2^3 \sin\theta_0}(A_2 + B_2)
\end{aligned} \tag{14}
$$

The positive directions of forces and moments, acting in the jointing area, are shown in fig.3.

Substituting the dependences (13) and (14) into the conjugation conditions (9) and (10) and taking into account the formulae for the momentless state (11) and (12), we'll receive the system of four algebraic equations for four constants of integration A_1, B_1, A_2 and B_2:

$$
\begin{cases}
-\dfrac{2-\mu}{2E\delta_1} pR_1^2 + A_1 = -\dfrac{1-\mu}{2E\delta_2} pR_2^2 \sin\theta_0 + A_2 \sin\theta_0 \\
\beta_1 (A_1 - B_1) = -\dfrac{\beta_2}{R_2}(A_2 - B_2) \\
2D_1 \beta_1^2 B_1 = 2D_2 \left(\dfrac{\beta_2}{R_2}\right)^2 B_2 \\
2D_1 \beta_1^3 (A_1 + B_1) = -\dfrac{1}{2} pR_2 \cos\theta_0 - \dfrac{2D_2}{\sin\theta_0}\left(\dfrac{\beta_2}{R_2}\right)^3 (A_2 + B_2)
\end{cases} \tag{15}
$$

Having solved the equations system (15) we determine the tension in the shells in the jointing area

$$
\begin{aligned}
\sigma_m &= \sigma_{m0} + \sigma_{mk} = \frac{N_m}{\delta} \pm \frac{6M_m}{\delta^2} \\
\sigma_t &= \sigma_{t0} + \sigma_{tk} = \frac{N_t}{\delta} \pm \frac{6M_t}{\delta^2}
\end{aligned} \tag{16}
$$

2 Numerical Results

The numerical computations were performed for the following constant values of shells parameters

$\mu = 0.3$, $E = 2 \times 10^5$ MPa, $R_1 = 13.7$ cm, $\delta_1 = \delta_2 = 0.7$ cm, $p = 5$ MPa

and some values of the radius of the spherical area $R_2 = 30$, 33 and 35 cm.

The computation results are shown in Tab. 1.

Table 1

R_2 / cm	β_1 / cm^{-1}	β_2	θ_0	$\sin\theta_0$	$\cos\theta_0$	B_2 / cm^{-1}	$\sigma_{m\max}$ / MPa
30	0.415	0.196	27°10′	0.167	0.986	−4.85	139
33	0.415	0.186	22°25′	0.412	0.924	−6.90	152
35	0.415	0.182	23°	0.391	0.900	−7.10	154

In such a way, the increase of the radius of the casing spherical area at the other constant parameters brings to the growth of meridional tensions in the dangerous cross- section (in tl conjugation area of the spherical and cylindrical parts).

The corresponding tensions in the conjugation area of the spherical part with the suction branch pipe (at $\theta_* = 9°$) considerably lower. For instance, at $R_2 = 30$ cm, $\sigma_{m\max}(\theta_*) = 111$ MPa.

The maximum stretching tensions increase on the inner surface in the conjugation area of the spherical part with the cylindrical one, and the maximum compressing tensions– on the outer surface in the conjugation area of the spherical part with the suction branch pipe.

The maximum tensions do not exceed the yield limit of the material ($\sigma_T = 200$ MPa), that is to say, the strength of the given casing is secured.

3 Conclusion

In such a way, the problem of the reliability estimation (safety factor–margin) of the power parts of the pump as a pressure vessel at the assigned structure dimensions, mechanic characteristics of the material and known loadings (at the effective pressure or numerically), using, for instance, method of finite elements.

The choice of this of that technique depends upon the project elaboration stage, and also upon technical and personnel properties of the design department.

So, in the stage of the preliminary designing, when one works up various ways and versions and varies a lot of parameters of different nature, it is more convenient to use the approximate analytical techniques. On the other hand, the use of numerical techniques at testing computation, when the structure is determined completely, its sizing, and the material is chosen.

The offered algorithm can be used both for the control- testing computations and in the stage of the preliminary designing.

The efficiency of use of the techniques can be estimated at its inculcation in computation practice.

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圆柱与圆球组合壳的压力容器强度分析

Js. S. 斯捷潘诺夫 V. A. 高尔东

（俄罗斯阿廖尔大学）

摘 要 研究致力于球带同两个圆柱结合壳的应变–变型状态分析，且模拟了液压泵管均匀载荷. 研究方法是解析的，所得结果的使用领域属于设计计算.

关键词 泵，壳，结合，拉伸，变形，强度

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