研究简报

Cardinal numbers of some sets (II)

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In considering some basic concepts of mathematics, we obtained some elementary results:

Theorem 1. Let X be an infinite set, $E(X)=\{R:R$ is an equivalence relation on $X\}$. $B(X)=X^2-E(X)$, then $|E(X)|=|B(X)|=2^{|X|}$

Theorem 2. Given two sets X and Y, Assume that $F_1(X, Y)=\{f: f\in Y^X \wedge f$ is surjective$\}$, $F_2(X,Y)=Y^X-F_1$ then $|F_1(X, Y)|=2^{|X|}$ $(|X| \geqslant \aleph_0,\ 2\leqslant |Y| \leqslant |X|)$

$|F_2(X, Y)|=|Y-F_1(X, Y)|=2^{|X|}$ $(|X|\geqslant \aleph_0.\ 3\leqslant |Y| \leqslant |X|)$

Theorem 3. If X and Y are infinite sets, Assume that $F_3=F_3(X,Y)=\{f:f\in Y^X \wedge f$ is injective$\}$, $F_4=Y^X-F_3$, then $|F_3|=|Y|^{|X|}$ $(|X| \leqslant |Y|)$, $|F_4|=|Y|^{|X|}$.

Theorem 4. If X is an infinite set, Assume that

$C=C(X)=\{f:f$ is a binary operation on $X\}$

$C_1=\{f:f\in C \wedge f$ is associative$\}$, $C_2=C-C_1$

then $|C_1|=|C_2|=|C|=2^{|X|}$.

Theorem 5. Let X be an infinite set, $C=C(X)=\{T:T$ is a topology on $X\}\subseteq P(P(X))$. Define C_1 $(\subsetneqq C)$ is a $(N\cdot H\cdot C\cdot)$-class if $(\forall T_1\in C_1)(\forall T_2\in C_1)(T_1\neq T_2 \rightarrow T_1$ and T_2 are not homeomorphic $\wedge$ T_1 and T_2 are incomparable), $\Sigma=\{C_1:C_1$ is a $(N.H.C.)$-class$\}$, then

$\max\{|C_1|:C_1\in\Sigma\}=2^{2^{|X|}}$, i.e. $|C_1|$ can reach the supremum.

Let X be an infinite set, $K=K(X)=\{A:A$ is an algebra (a field) of sets on $X\}\subseteq P(P(X))$. Define $K_1(\subsetneqq K)$is a $(N.I.)$-class, if $(\forall A_1\in K_1)(\forall A_2\in K_1)(A_1\neq A_2 \rightarrow A_1$ and A_2 are not isomorphic), $\Sigma=\{K_1:K_1$ is a $(N.I.)$-class$\}$, then $\max\{|K_1|:K_1\in\Sigma\}=2^{2^{|X|}}$, i. e. $|K_1|$ can reach the supremum.

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