区域外层具有奇性的Stiff问题的渐近分析

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【摘要】 将区域外层具有奇性的Stiff问题（A）化为泛函的变分问题（B），然后利用Lions引入的渐近展开式讨论（B）的解，即设（B）的解是ε的幂级数展开式：

$$u_\varepsilon=\frac{u^{-1}}{\varepsilon}+u^0+\varepsilon u^1+\varepsilon^2u^2+\cdots$$

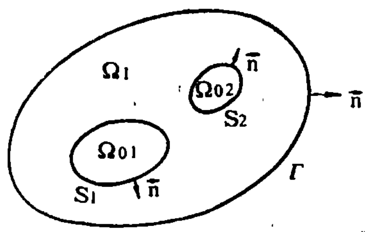
将u_ε代入（B）后求得系数u^{-1}，u^0，u^1，u^2，…的一系列循环公式.

关键词：奇性，Stiff（硬的），渐近展开式.

一、问题的提出

本文考虑区域外层具有奇性的Stiff问题（A）.

设Ω是$\mathbf{R}^n$中的一个开集，$\Omega=\Omega_{01}+\Omega_{02}+\Omega_1$，其边界分别为$S_1$，$S_2$，$\Gamma$，如图所示.



$$(A):\begin{cases}-\sum\limits_{i=1}^{n}\dfrac{\partial}{\partial x_i}\left[b_{01}(x)\dfrac{\partial u}{\partial x_i}\right]=f_{01},\quad f_{01}\in L^2(\Omega_{01}), & (1.1)\\ -\sum\limits_{i=1}^{n}\dfrac{\partial}{\partial x_i}\left[b_{02}(x)\dfrac{\partial u}{\partial x_i}\right]=f_{02},\quad f_{02}\in L^2(\Omega_{02}), & (1.2)\\ \varepsilon\left(-\sum\limits_{i=1}^{n}\dfrac{\partial}{\partial x_i}\left[b_1(x)\dfrac{\partial u}{\partial x_i}\right]+m(x)u\right)=f_1,\quad f_1\in L^2(\Omega_1), & (1.3)\\ u_{01}|_{S_1}=u_1|_{S_1}, & (1.4)\end{cases}$$

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$$\left\{\begin{aligned}&u_{02}|_{s_2}=u_1|_{s_2}, &(1.5)\\&b_{01}(x)\frac{\partial u_{01}}{\partial n}\Big|_{s_1}=\varepsilon b_1(x)\frac{\partial u_1}{\partial n}\Big|_{s_1}, &(1.6)\\&b_{02}(x)\frac{\partial u_{02}}{\partial n}\Big|_{s_2}=\varepsilon b_1(x)\frac{\partial u_1}{\partial n}\Big|_{s_2}, &(1.7)\\&u_1|_\Gamma=0. &(1.8)\end{aligned}\right.$$

其中，$f=[f_{01}, f_{02}, f_1]$，f_{01}，f_{02}，f_1分别是f在Ω_{01}，Ω_{02}，Ω_1上的限制；$u=[u_{01}, u_{02}, u_1]$，u_{01}，u_{02}，u_1分别是u在Ω_{01}，Ω_{02}，Ω_1上的限制．$0<\alpha\leqslant b_{01}$（及b_{02}，b_1）$\leqslant\beta$，$0\leqslant m(x)\leqslant\beta_0$．问题（$A$）中的所有导数都是分布意义下的．本文中的$C$表示与$\varepsilon$无关的常数．问题（$A$）的$\varepsilon$分布在区域外层$\Omega_1$上，区域外层具有奇性比区域内层具有奇性的情况要复杂，前者会引起整个Ω域上的ε^{-1}阶的奇性，而后者仅在内层引起ε^{-1}阶的奇性．

我们引入：

$$a_{01}(u,v)=\int_{\Omega_{01}}\sum_{i=1}^n b_{01}(x)\frac{\partial u}{\partial x_i}\frac{\partial v}{\partial x_i}dx,$$

$$a_{02}(u,v)=\int_{\Omega_{02}}\sum_{i=1}^n b_{02}(x)\frac{\partial u}{\partial x_i}\frac{\partial v}{\partial x_i}dx,$$

$$a_1(u,v)=\int_{\Omega_1}\left[\sum_{i=1}^n b_1(x)\frac{\partial u}{\partial x_i}\frac{\partial v}{\partial x_i}+m(x)uv\right]dx,$$

$$(f,v)=\int_\Omega fv\,dx=\int_{\Omega_{01}}f_{01}v\,dx+\int_{\Omega_{02}}f_{02}v\,dx+\int_{\Omega_1}f_1v\,dx.$$

考虑问题（B）：求$u\in H_0^1(\Omega)$，使得

（B）：　$a_{01}(u,v)+a_{02}(u,v)+\varepsilon a_1(u,v)=(f,v)$

对任意的$v\in H_0^1(\Omega)$成立．

定理1.1. 问题（A）与问题（B）是等价的．

证． 先证（B）成立时，（A）也成立．

在（B）式中，若取$v=\varphi\in\mathscr{D}(\Omega_{01})$，有(1.1)式；若取$v=\varphi\in\mathscr{D}(\Omega_{02})$，有(1.2)式；若取$v=\varphi\in\mathscr{D}(\Omega_1)$，有(1.3)式．

在Ω_1上应用Green公式，并利用(1.3)式，有

$$\varepsilon a_1(u,v)=-\varepsilon\int_{s_1}b_1(s)\frac{\partial u}{\partial n}v\,ds-\varepsilon\int_{s_2}b_1(s)\frac{\partial u}{\partial n}v\,ds+\int_{\Omega_1}f_1v\,dx, \tag{1.9}$$

在Ω_{01}上应用Green公式，并利用（1.1），有

$$a_{01}(u,v)=\int_{s_1} b_{01}(s)\frac{\partial u}{\partial n}v\,ds+\int_{\Omega_{01}} f_{01}v\,dx, \tag{1.10}$$

在Ω_{02}上应用Green公式，并利用（1.2），有

$$a_{02}(u,v)=\int_{s_2} b_{02}(s)\frac{\partial u}{\partial n}v\,ds+\int_{\Omega_{02}} f_{02}v\,dx. \tag{1.11}$$

将(1.9)—(1.11)式代入（B）式，并利用v的任意性有(1.6)及(1.7)式.

∵　$u_1\in H_0^1(\Omega)$，在边界Γ上有$u_1|_\Gamma=0$，即(1.8)式成立. 又由[2]的迹性质定理，有$u_{01}|_{s_1}=u_1|_{s_1}$，$u_{02}|_{s_2}=u_1|_{s_2}$，即（1.4），(1.5）式成立. 故问题（A）成立.

应用文献[2]中的Green公式，可证问题（A）成立时，问题（B）也成立.

定理1.2.　问题（B）存在唯一解.

证. 设$A(u,v)=a_{01}(u,v)+a_{02}(u,v)+\varepsilon a_1(u,v)$，显然$A(u,v)$具有双线性及有界性.

又由Poincarè—Fridrich不等式，有$A(v,v)\geqslant\varepsilon\alpha\|v\|^2_{H'(\Omega)}$.

又(f,v)是$H_0'(\Omega)$上的有界加法泛函，由文献[3]知，问题（B）存在唯一解.

二、基　本　引　理

设$V=\{v\,|\,v\in H'_{(\Omega)},\ v|_{\Omega_{01}}=r,\ v|_{\Omega_{02}}=k,\ v|_\Gamma=0,\ \forall r,\ k\in R^1\}$，$0<\alpha\leqslant b_1(x)\leqslant\beta$，$0\leqslant m(x)\leqslant\beta_0$.

考虑问题：求$u\in V$，$\forall v\in V$，使

$$a_1(u,v)=(f,v) \tag{2.1}$$

成立.

基本引理2.1.　问题（2.1）存在唯一解$u\in V$，且满足：

$$\begin{cases}-\sum\limits_{i=1}^{n}\dfrac{\partial}{\partial x_i}\left[b_1(x)\dfrac{\partial u}{\partial x_i}\right]+m(x)u=f_1, & (2.2)\\ u|_{s_1}=r,\\ u|_{s_2}=k,\\ u|_\Gamma=0,\\ -\displaystyle\int_{s_1} b_1(s)\frac{\partial u}{\partial n}ds=\int_{\Omega_{01}} f_{01}dx,\\ -\displaystyle\int_{s_2} b_1(s)\frac{\partial u}{\partial n}ds=\int_{\Omega_{02}} f_{02}dx.\end{cases}$$

（2.2）式是一种有限制条件的第一边值问题.

证.　由定义知，$a_1(u,v)$是V上的对称有界双线性形式，(f,v)是V上的有界加法泛

函.取$v\in V$，对v在Ω_1上应用文献[2]的迹定理，有

$$\left[\int_{S_1} r^2 ds\right]^{\frac{1}{2}} \leqslant C\|v\|_{H'(\Omega_1)}, \quad 即\ r\leqslant C\|v\|_{H'(\Omega_1)}, \tag{2.3}$$

$$\left[\int_{S_2} k^2 ds\right]^{\frac{1}{2}} \leqslant C\|v\|_{H'(\Omega_1)}, \quad 即\ k\leqslant C\|v\|_{H'(\Omega_1)}. \tag{2.4}$$

$\forall v\in V$，由(2.3)及(2.4)式，有

$$\begin{aligned}\|v\|^2_{H'(\Omega)} &= \|v\|^2_{H'(\Omega_1)} + \|v\|^2_{H'(\Omega_{01})} + \|v\|^2_{H'(\Omega_{02})} \\ &= \|v\|^2_{H'(\Omega_1)} + \int_{\Omega_{01}} r^2 dx + \int_{\Omega_{02}} k^2 dx \\ &\leqslant \|v\|^2_{H'(\Omega_1)} + C\|v\|^2_{H'(\Omega_1)} + C\|v\|^2_{H'(\Omega_1)} \\ &\leqslant C\|v\|^2_{H'(\Omega_1)}.\end{aligned} \tag{2.5}$$

由于$v\in H'(\Omega_1)$，$v|_\Gamma=0$，$meas(\Gamma)>0$．又由文献[4]中的Deny—Lions不等式，有

$$\|\nabla v\|_{L^2(\Omega_1)} \geqslant C\|v\|_{H'(\Omega_1)}, \tag{2.6}$$

因此，由(2.5)及(2.6)式，有

$$\begin{aligned}a_1(v,v) &= \int_{\Omega_1}[b_1(x)\nabla v\cdot\nabla v + m(x)v^2]dx \\ &\geqslant \alpha\int_{\Omega_1}[\nabla v\cdot\nabla v + m(x)\cdot v^2]dx \\ &\geqslant \alpha\|v\|_{H'(\Omega)}\end{aligned} \tag{2.7}$$

由(2.7)式和文献[3]知，问题(2.1)存在唯一解$u\in V$．

基本引理2.1在区域外层具有奇性的Stiff问题（A）的渐近展开式中，利用循环公式确定展开式的系数时起重要的作用。

三、解的渐近展开

定理3.1. 问题（B）的解可由渐近展开式$u_\varepsilon=\dfrac{u^{-1}}{\varepsilon}+u^0+\varepsilon u^1+\varepsilon^2u^2+\cdots$来求解，且$u^{-1}$，$u^0$，$u^j$（$j\geqslant1$）由如下关系式确定．

$$u^{-1}=[u^{-1}_{01},\ u^{-1}_{02},\ u^{-1}_1]$$

其中

$$u^{-1}_{01}=r, \tag{3.1a}$$

$$u^{-1}_{02}=k, \tag{3.1b}$$

u_1^{-1}:

$$\begin{cases} -\sum_{i=1}^{n} \frac{\partial}{\partial x_i}\left[b_1(x)\frac{\partial u_1^{-1}}{\partial x_i}\right] + m u_1^{-1} = f_1, \\ u_1^{-1}|s_1 = r, u_1^{-1}|s_2 = k, u_1^{-1}|_\Gamma = 0, \\ -\int_{s_1} b_1(s)\frac{\partial u_1^{-1}}{\partial n} ds = \int_{\Omega_{01}} f_{01} dx, \\ -\int_{s_2} b_1(s)\frac{\partial u_1^{-1}}{\partial n} ds = \int_{\Omega_{02}} f_{02} dx; \end{cases} \tag{3.1c}$$

$u^0 = [u_{01}^0,\ u_{02}^0,\ u_1^0]$.

其中　u_{01}^0:

$$\begin{cases} -\sum_{i=1}^{n} \frac{\partial}{\partial x_i}\left[b_{01}(x)\frac{\partial u_{01}^0}{\partial x_i}\right] = f_{01}, \\ b_{01}(s)\frac{\partial u_{01}^0}{\partial n}\Big|_{s_1} = b_1(s)\frac{\partial u_1^{-1}}{\partial n}\Big|_{s_1}, \end{cases} \tag{3.2a}$$

u_{02}^0:

$$\begin{cases} -\sum_{i=1}^{n} \frac{\partial}{\partial x_i}\left[b_{02}(x)\frac{\partial u_{02}^0}{\partial x_i}\right] = f_{02}, \\ b_{02}(s)\frac{\partial u_{02}^0}{\partial n}\Big|_{s_2} = b_1(s)\frac{\partial u_1^{-1}}{\partial n}\Big|_{s_2}, \end{cases} \tag{3.2b}$$

u_1^0:

$$\begin{cases} -\sum_{i=1}^{n} \frac{\partial}{\partial x_i}\left[b_1(x)\frac{\partial u_1^0}{\partial x_i}\right] = 0, \\ u_1^0|s_1 = u_{01}^0|s_1, \\ u_1^0|s_2 = u_{02}^0|s_2, \\ \int_{s_1} b_1(s)\frac{\partial u_1^0}{\partial n} ds = 0, \\ \int_{s_2} b_1(s)\frac{\partial u_1^0}{\partial n} ds = 0, \\ u_1^0|_\Gamma = 0; \end{cases} \tag{3.2c}$$

$u^j = [u_{01}^j,\ u_{02}^j,\ u_1^j]$.

其中　u_{01}^j:

$$\begin{cases} -\sum_{i=1}^{n} \frac{\partial}{\partial x_i}\left[b_{01}(x)\frac{\partial u_{01}^j}{\partial x_i}\right] = 0, \\ b_{01}(s)\frac{\partial u_{01}^j}{\partial n}\Big|_{s_1} = b_1(s)\frac{\partial u_1^{j-1}}{\partial n}\Big|_{s_1}, \end{cases} \tag{3.3a}$$

u_{02}^j:

$$\begin{cases} -\sum_{i=1}^{n} \frac{\partial}{\partial x_i}\left[b_{02}(x)\frac{\partial u_{02}^j}{\partial x_i}\right] = 0, \\ b_{02}(s)\frac{\partial u_{02}^j}{\partial n}\Big|_{s_2} = b_1(s)\frac{\partial u_1^{j-1}}{\partial n}\Big|_{s_2}, \end{cases} \tag{3.3b}$$

$$u_1^j:\begin{cases} -\sum_{i=1}^{n}\dfrac{\partial}{\partial x_i}\left[b_1(x)\dfrac{\partial u_1^j}{\partial x_i}\right]=0,\\ u_1^j|_{s_1}=u_{01}^j|_{s_1},\\ u_1^j|_{s_2}=u_{02}^j|_{s_2},\\ u_1^j|_{\Gamma}=0,\\ \int_{s_1} b_1(s)\dfrac{\partial u_1^j}{\partial n}\mathrm{d}s=0,\\ \int_{s_2} b_1(s)\dfrac{\partial u_1^j}{\partial n}\mathrm{d}s=0. \end{cases} \tag{3.3c}$$

其中$j=1, 2, \cdots$.

证. 将 $u_\varepsilon=\dfrac{u^{-1}}{\varepsilon}+u^0+\varepsilon u'+\cdots$代入问题($B$),并比较$\varepsilon$同次幂的系数得:

$$a_{01}(u^{-1},v)+a_{02}(u^{-1},v)=0, \tag{3.4}$$

$$a_{01}(u^0,v)+a_{02}(u^0,v)+a_1(u^{-1},v)=(f,v), \tag{3.5}$$

$$a_{01}(u^j,v)+a_{02}(u^j,v)+a(u^{j-1},v)=0,\quad j=1,2,\cdots. \tag{3.6}$$

引入限制集合:

$$Y_0=\{v\,|\,v\in H_0^1(\Omega),\ v|_{\rho_{01}}=v_{01}=r,\ v|_{\rho_{02}}=v_{02}=k,\ \forall r,k\in\mathbf{R}'\}, \tag{3.7}$$

由Y_0的定义,不难证明$\forall u\in Y_0$的充要条件是

$$a_{01}(u,v)+a_{02}(u,v)=0,\quad \forall v\in H_0^1(\Omega). \tag{3.8}$$

对于u^{-1}.

已知u^{-1}满足(3.4)及(3.5)式. 由(3.4)式可知,$u^{-1}\in Y_0$,在(3.5)式中取$v\in Y_0$,就有

$$a_1(u^{-1},v)=(f,v), \tag{3.9}$$

由基本引理2.1知,存在唯一的$u^{-1}\in Y_0$,使(3.9)式成立.

取$v\in\mathscr{D}(\Omega_1)$,得

$$-\sum_{i=1}^{n}\frac{\partial}{\partial x_i}\left[b_1(x)\frac{\partial u_1^{-1}}{\partial x_i}\right]+m(x)u_1^{-1}=f_1, \tag{3.10}$$

$\forall v\in H_0^1(\Omega_1)$,在$\Omega_1$上应用Green公式,有

$$a_1(u^{-1},v)=(f_1,v)-\int_{s_1}b_1(s)\frac{\partial u_1^{-1}}{\partial n}v\,\mathrm{d}s-\int_{s_2}b_1(s)\frac{\partial u_1^{-1}}{\partial n}v\,\mathrm{d}s, \tag{3.11}$$

将(3.11)式代入(3.9)式,得$\forall v\in Y_0$,

$$-\int_{s_1}b_1(s)\frac{\partial u_1^{-1}}{\partial n}v\,\mathrm{d}s-\int_{s_2}b_1(s)\frac{\partial u_1^{-1}}{\partial n}v\,\mathrm{d}s=(f_{01},v)+(f_{02},v),$$

由r,k的任意性,有

$$-\int_{s_1}b_1(s)\frac{\partial u_1^{-1}}{\partial n}\mathrm{d}s=\int_{\Omega_{01}}f_{01}\,\mathrm{d}x,\qquad -\int_{s_2}b_1(s)\frac{\partial u_1^{-1}}{\partial n}\mathrm{d}s=\int_{\Omega_{02}}f_{02}\,\mathrm{d}x.$$

即u^{-1}满足(3.1)式.

对于u^0.

已知u^0满足：

$$\begin{cases} a_{01}(u^0,v)+a_{02}(u^0,v)+a_1(u^{-1},v)=(f,v), & (3.12)\\ a_{01}(u^1,v)+a_{02}(u^1,v)+a_1(u^0,v)=0. & (3.13)\end{cases}$$

在(3.12)中分别取$v\in\mathscr{D}(\Omega_{01})$及$v\in\mathscr{D}(\Omega_{02})$，有

$$-\sum_{i=1}^{n}\frac{\partial}{\partial x_i}\left[b_{01}(x)\frac{\partial u_{01}^0}{\partial x_i}\right]=f_{01},\qquad -\sum_{i=1}^{n}\frac{\partial}{\partial x_i}\left[b_{02}(x)\frac{\partial u_{02}^0}{\partial x_i}\right]=f_{02}.$$

$\forall\, v\in H_0^1(\Omega)$，在$\Omega_{01}$及$\Omega_{02}$上分别用Green公式，有

$$a_{01}(u^0,v)=\int_{\Omega_1}b_1(s)\frac{\partial u_{01}^0}{\partial n}v\,ds+\int_{\Omega_{01}}f_{01}v\,\mathrm{d}x, \tag{3.14}$$

$$a_{02}(u^0,v)=\int_{s_2}b_2(s)\frac{\partial u_{02}^0}{\partial n}v\,ds+\int_{\Omega_{02}}f_{02}v\,\mathrm{d}x. \tag{3.15}$$

将(3.11)、(3.14)、(3.15)式代入(3.5)式，并由v的任意性，有

$$b_{01}(s)\frac{\partial u_{01}^0}{\partial n}\bigg|_{s_1}=b_1(s)\frac{\partial u_1^{-1}}{\partial n}\bigg|_{s_1},\qquad b_{02}(s)\frac{\partial u_{02}^0}{\partial n}\bigg|_{s_2}=b_1(s)\frac{\partial u_1^{-1}}{\partial n}\bigg|_{s_2},$$

即u_{01}^0，u_{02}^0分别满足(3.2a)、(3.2b).

(3.2a)，(3.2b)是Neumann问题，由文献[5]的第二章知，这两个问题的解相差一个常数，这个常数则由u_1^0满足的方程唯一确定.

在(3.13)式中取$v\in Y_0$，得

$$a_1(u^0,v)=0, \tag{3.16}$$

取$v=\varphi\in\mathscr{D}(\Omega_1)$，有

$$-\sum_{i=1}^{n}\frac{\partial}{\partial x_i}\left[b_1(x)\frac{\partial u_1^0}{\partial x_i}\right]=0, \tag{3.17}$$

由$u^0\in H_0^1(\Omega)$及文献[2]的迹定理，有

$$u_1^0|_{s_1}=u_{01}^0|_{s_1},\quad u_1^0|_{s_2}=u_{02}^0|_{s_2}. \tag{3.18}$$

由(3.16)式，应用Green公式和类似u_1^{-1}的推导可得

$$\int_{s_1}b_1(s)\frac{\partial u_1^0}{\partial n}\mathrm{d}s=0,\qquad \int_{s_2}b_1(s)\frac{\partial u_1^0}{\partial n}\mathrm{d}s=0. \tag{3.19}$$

由(3.16)—(3.19)式，得(3.2c)式.

同理，可以证明u^j满足(3.3a)—(3.3c)式，$j=1,2,\cdots$.

四、误差估计

定理4.1. 问题(B)成立估计式：

$$\|u_\varepsilon-\left(\frac{u^{-1}}{\varepsilon}+u^0+\varepsilon u^1+\cdots+\varepsilon^j u^j\right)\|_{H_0^1(\Omega)}\leqslant C\varepsilon^{j+1}\|u^{j+1}\|_{H_0^1(\Omega)} \tag{4.1}$$

证. 设$u_\varepsilon^j=\dfrac{u^{-1}}{\varepsilon}+u^0+\varepsilon u^1+\cdots+\varepsilon^j u^j$,

$$u_\varepsilon^{j+1}=u_\varepsilon^j+\varepsilon^{j+1}u^{j+1},$$

$$W_\varepsilon=u_\varepsilon-u_\varepsilon^{j+1},$$

$$\Pi_\varepsilon(u,v)=a_{01}(u,v)+a_{02}(u,v)+\varepsilon a_1(u,v),$$

由$u^j(j\geqslant-1)$的定义和直接计算有

$$\Pi_\varepsilon(u_\varepsilon^{j+1},v)=(f,v)+\varepsilon^{j+2}a_1(u^{j+1},v), \tag{4.2}$$

$$\Pi_\varepsilon(u_\varepsilon,v)=(f,v). \tag{4.3}$$

(4.3)式减(4.2)式，有

$$\Pi_\varepsilon(w_\varepsilon,v)=-\varepsilon^{j+2}a_1(u^{j+1},v),$$

取$v=w_\varepsilon$，有

$$\Pi_\varepsilon(w_\varepsilon,w_\varepsilon)=-\varepsilon^{j+2}a_1(u^{j+1},w_\varepsilon),$$

$$\|w_\varepsilon\|_{H_0^1(\Omega)}\leqslant C\varepsilon^{j+1}\|u^{j+1}\|_{H_0^1(\Omega)}. \tag{4.4}$$

$$\begin{aligned}\therefore\quad \|u_\varepsilon-u_\varepsilon^j\|_{H_0^1(\Omega)}&=\|u_\varepsilon-u_\varepsilon^{j+1}+\varepsilon^{j+1}u^{j+1}\|_{H_0^1(\Omega)}\\&\leqslant\|u_\varepsilon-u_\varepsilon^{j+1}\|_{H_0^1(\Omega)}+\varepsilon^{j+1}\|u^{j+1}\|_{H_0^1(\Omega)}\\&\leqslant C\varepsilon^{j+1}\|u^{j+1}\|_{H_0^1(\Omega)}.\end{aligned}$$

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Asymptotical analysis of the Stiff problem with Singularity on the exterior domain

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Abstract: In this paper the Stiff problem (A) with Singularity on the exterior domain is reduced to the functional Variational problem(B).

Using an asymptotical expansion introduced by Lions, we then discuss the Solution of (B), i.e., letting the solution of (B) be a power series expansion in ε:

$$u_\varepsilon=\frac{u^{-1}}{\varepsilon}+u^0+\varepsilon u^1+\varepsilon^2 u^2+\cdots$$

and Substituting u_ε into (B) , is obtoined a sequence of cyclic formulas of coefficients u^{-1}, u^0, u^1, u^2, ….