具有放牧率及扩散的概周期竞争模型

王长有[1,2,3]，王 术[1]，李林锐[1]
（1.北京工业大学 应用数理学院，北京 100124；2.重庆邮电大学 工业物联网与网络化控制教育部重点实验室，重庆 400065；3.重庆邮电大学 应用数学研究所，重庆 400065）

摘 要：研究了一类具有放牧率及扩散的3种群概周期竞争模型，利用上、下解方法，Schauder不动点定理以及Lyapunov稳定性理论，得到了确保该模型空间齐次概周期解的存在性及稳定性的充分条件，推广了已有的相应结果.

关键词：概周期解；稳定性；竞争模型；放牧率；上、下解方法
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Almost Periodic Competition Models With Grazing Rates and Diffusions

WANG Chang-you[1,2,3], WANG Shu[1], LI Lin-rui[1]
(1. College of Applied Sciences, Beijing University of Technology, Beijing 100124, China;
2. Key Laboratory of Industrial Internet of Things & Networked Control of Ministry of Education, Chongqing University of Posts and Telecommunications, Chongqing 400065, China;
3. Institute of Applied Mathematics, Chongqing University of Posts and Telecommunications, Chongqing 400065, China)

Abstract: Almost periodic solution of a three-species competition system with grazing rates and diffusions were investigated by using the method of upper and lower solutions, the Schauder fixed point theorem as well as Lyapunov stability theory. Sufficient conditions guaranteeing the existence and stability of the strictly positive space homogenous almost periodic solution of the system were obtained, and some known results were improved.

Key words: almost periodic solution; stability; competition model; grazing rates; method of upper and lower solutions

1 模型介绍

考虑具有放牧率及扩散的3种群概周期竞争模型

$$\begin{cases} \dfrac{\partial v_1(x,t)}{\partial t} = k_1(t)\Delta v_1(x,t) + v_1(x,t)[a_1(t) - b_1(t)v_1(x,t) - c_1(t)v_2(x,t) - d_1(t)v_3(x,t) + \\ \quad f_1(t), \ (x,t) \in \Omega \times \mathbb{R}^+ \\ \dfrac{\partial v_2(x,t)}{\partial t} = k_2(t)\Delta v_2(x,t) + v_2(x,t)[a_2(t) - b_2(t)v_1(x,t) - c_2(t)v_2(x,t) - d_2(t)v_3(x,t) + \\ \quad f_2(t), \ (x,t) \in \Omega \times \mathbb{R}^+ \\ \dfrac{\partial v_3(x,t)}{\partial t} = k_3(t)\Delta v_3(x,t) + v_3(x,t)[a_3(t) - b_3(t)v_1(x,t) - c_3(t)v_2(x,t) - d_3(t)v_3(x,t) + \\ \quad f_3(t), \ (x,t) \in \Omega \times \mathbb{R}^+ \end{cases} \quad (1)$$

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作者简介：王长有(1968—)，男，教授，主要从事微分方程理论及应用方面的研究，E-mail:wangcy@cqupt.edu.cn.

式中：$k_i(t)$，$a_i(t)$，$b_i(t)$，$c_i(t)$，$d_i(t)$，$f_i(t)$ $(i=1,2,3)$均为实数集$\mathbb{R}$上的概周期函数，分别表示种群之间的扩散率、竞争率及放牧率；Δ 表示 Ω 上的 Laplace 算子；$(x,t)\in\Omega\times\mathbb{R}^+$，$\Omega$ 为 3 种群的栖息区域，它为$\mathbb{R}^n$中的具有光滑边界$\partial\Omega$的有界开集. 式(1)相应的边界条件及初始条件分别为

$$\frac{\partial v_i(x,t)}{\partial n}=0,\ i=1,2,3,\ (x,t)\in\partial\Omega\times\mathbb{R}^+ \quad (2)$$

$$v_i(x,0)=v_{i0}(x)\geqslant 0,\ 且不恒为0,\ i=1,2,3,x\in\overline{\Omega} \quad (3)$$

式中：$\partial/(\partial n)$表示外法向导数；$v_i(x,t)$表示第 i 种群在点 $x=(x_1,\cdots,x_n)$和时刻 t 的密度.

系统(1)～(3)描述了 3 种群之间的相互作用，是生物数学中的一种重要的数学模型，吸引了广大科技工作者对其进行深入广泛的研究[1-5]. 当种群均匀分布即无扩散时，姜东平等[1-2]讨论了与上述方程对应的 2 种群模型在系数分别为周期函数和概周期函数时的周期解和概周期解的存在性、唯一性及稳定性. 陈风德等[3]将文献[2]的结果推广到 n 种群情形. 当上述系统无放牧率时，Pao 等[4]利用上、下解方法对上述模型的常数平衡态的稳定性进行了研究. Liu 等[5]研究了具有放牧率及扩散的 n 种群竞争模型周期解的稳定性. 然而，一般来讲，种群所依赖的环境不一定严格按周期规律变化，有时按概周期规律变化，因此研究既含扩散项又含放牧率的多种群概周期生态模型具有重要意义. 本文对具有放牧率及扩散的 3 种群竞争模型(1)～(3)的概周期解进行讨论，利用偏微分方程的上、下解方法，Schauder 不动点定理以及 Lypunov 第二稳定性理论获得了确保该模型空间齐次概周期解的存在性和稳定性的充分条件，推广了文献[2-3，5]的结果. 关于周期解及概周期解的研究还可参考文献[6－9].

2 上、下解的构造

为了证明本文的主要结果，先介绍几个定义及引理.

定义 1 对实数集$\mathbb{R}$上的连续函数$f(t)$，若对任意 $\varepsilon>0$ 存在 $T(\varepsilon)>0$，使得任一长度为 $T(\varepsilon)$ 的区间中至少有一点 τ，使得$|f(t+\tau)-f(t)|\leqslant\varepsilon$ 对任意 $t\in\mathbb{R}$ 成立，则称$f(t)$是概周期函数.

定义 2 若光滑函数 $u(t)=(v_1(t),v_2(t),v_3(t))$在$\mathbb{R}^+$上满足方程(1)且 $u(t)$是概周期的，则称 $u(t)$为方程(1)的空间齐次概周期解，记为 $u(t,T(\varepsilon))$.

定义 3 若系统(1)及相应的边界条件(2)对任意给定的非负光滑初值

$$u(x,0)=(v_1(x,0),v_2(x,0),v_3(x,0))=(v_{10}(x),v_{20}(x),v_{30}(x))\geqslant 0,\ 且不恒为0,\ x\in\Omega$$

存在唯一正解 $u(x,t)=(v_1(x,t),v_2(x,t),v_3(x,t))$，且有$\lim\limits_{t\to\infty}(u_i(x,t)-u_i(t,T(\varepsilon)))=0,i=1,2,3$，关于 $x\in\overline{\Omega}$ 一致成立，则称空间齐次概周期解 $u(t,T(\varepsilon))$是全局稳定的.

定义 4 设 $\overline{V}(x,t)\equiv(\bar{v}_1(x,t),\bar{v}_2(x,t),\bar{v}_3(x,t))$，$\underline{V}(x,t)=(\underline{v}_1(x,t),\underline{v}_2(x,t),\underline{v}_3(x,t))$，如果 $\overline{V}(x,t)\geqslant\underline{V}(x,t)$，并满足

$$\begin{cases}\dfrac{\partial\bar{v}_1(x,t)}{\partial t}\geqslant k_1(t)\Delta\bar{v}_1(x,t)+\bar{v}_1(x,t)[a_1(t)-\\ \quad b_1(t)\bar{v}_1(x,t)-c_1(t)\underline{v}_2(x,t)-d_1(t)\underline{v}_3(x,t)]+\\ \quad f_1(t),\ (x,t)\in\Omega\times\mathbb{R}^+\\ \dfrac{\partial\bar{v}_2(x,t)}{\partial t}\geqslant k_2(t)\Delta\bar{v}_2(x,t)+\bar{v}_2(x,t)[a_2(t)-\\ \quad b_2(t)\underline{v}_1(x,t)-c_2(t)\bar{v}_2(x,t)-d_2(t)\underline{v}_3(x,t)]+\\ \quad f_2(t),\ (x,t)\in\Omega\times\mathbb{R}^+\\ \dfrac{\partial\bar{v}_3(x,t)}{\partial t}\geqslant k_3(t)\Delta\bar{v}_3(x,t)+\bar{v}_3(x,t)[a_3(t)-\\ \quad b_3(t)\underline{v}_1(x,t)-c_3(t)\underline{v}_2(x,t)-d_3(t)\bar{v}_3(x,t)]+\\ \quad f_3(t),\ (x,t)\in\Omega\times\mathbb{R}^+\end{cases} \quad (4)$$

$$\frac{\partial\bar{v}_i(x,t)}{\partial n}\geqslant 0,\quad i=1,2,3,\quad (x,t)\in\partial\Omega\times\mathbb{R}^+ \quad (5)$$

$$\bar{v}_i(x,0)\geqslant v_{i0}(x),\ i=1,2,3,\ x\in\overline{\Omega} \quad (6)$$

$$\begin{cases}\dfrac{\partial\underline{v}_1(x,t)}{\partial t}\leqslant k_1(t)\Delta\underline{v}_1(x,t)+\underline{v}_1(x,t)[a_1(t)-\\ \quad b_1(t)\underline{v}_1(x,t)-c_1(t)\bar{v}_2(x,t)-d_1(t)\bar{v}_3(x,t)]+\\ \quad f_1(t),\ (x,t)\in\Omega\times\mathbb{R}^+\\ \dfrac{\partial\underline{v}_2(x,t)}{\partial t}\leqslant k_2(t)\Delta\underline{v}_2(x,t)+\underline{v}_2(x,t)[a_2(t)-\\ \quad b_2(t)\bar{v}_1(x,t)-c_2(t)\underline{v}_2(x,t)-d_2(t)\bar{v}_3(x,t)]+\\ \quad f_2(t),\ (x,t)\in\Omega\times\mathbb{R}^+\\ \dfrac{\partial\underline{v}_3(x,t)}{\partial t}\leqslant k_3(t)\Delta\underline{v}_3(x,t)+\underline{v}_3(x,t)[a_3(t)-\\ \quad b_3(t)\bar{v}_1(x,t)-c_3(t)\bar{v}_2(x,t)-d_3(t)\underline{v}_3(x,t)]+\\ \quad f_3(t),\ (x,t)\in\Omega\times\mathbb{R}^+\end{cases} \quad (7)$$

$$\frac{\partial \underline{v}_i(x,t)}{\partial n} \leqslant 0, \quad i=1,2,3, \quad (x,t)\in \delta\Omega\times\mathbb{R}^+ \tag{8}$$

$$\underline{v}_i(x,0) \leqslant v_{i0}(x),\ i=1,2,3,\ x\in\overline{\Omega} \tag{9}$$

则称 $\overline{V}(x,t)$、$\underline{V}(x,t)$ 分别为系统(1)~(3)的一对有序上、下解.

引理 1[10]　设 $\overline{V}(x,t)$、$\underline{V}(x,t)$ 分别为系统(1)~(3)的一对有序上、下解，则系统(1)~(3)存在唯一解 $V(x,t)$，且 $\overline{V}(x,t)\geqslant V(x,t)\geqslant \underline{V}(x,t)$.

对 $\mathbb{R}$ 上的概周期函数 $F(t)$，记

$$\widetilde{F}=\sup\{F(t),t\in\mathbb{R}\},\ \underset{\sim}{F}=\inf\{F(t),t\in\mathbb{R}\}$$

$$M[F]=\lim_{t-s\to\infty}\left\{\int_s^t F(\tau)\,\mathrm{d}\tau/(t-s)\right\}$$

当 $F(t)$ 是 T 一周期函数时，$M[F]=\int_0^T F(s)\,\mathrm{d}s/T$.

引理 2[11]（Schauder 不动点定理）　假设 K 是 Banach 空间 E 的有界闭凸集，而 $T: K\to K$ 是紧映射，则存在 $x^*\in K$，使得 $Tx^*=x^*$.

3　主要结果及证明

定理 1　若 $\underset{\sim}{a_i},\underset{\sim}{b_i},\underset{\sim}{c_i},\underset{\sim}{d_i},\underset{\sim}{f_i}$ 是正数，且对 $i=1,2,3$ 有

$$(\widetilde{b}_i+\widetilde{c}_i+\widetilde{d}_i)/\underset{\sim}{a_i}\leqslant L=\min\{\sqrt{\underset{\sim}{b_1}/\widetilde{f}_1},\sqrt{\underset{\sim}{c_2}/\widetilde{f}_2},\sqrt{\underset{\sim}{d_3}/\widetilde{f}_3},(\underset{\sim}{d_1}+\underset{\sim}{c_1})/\widetilde{a}_1,(\underset{\sim}{b_2}+\underset{\sim}{d_2})/\widetilde{a}_2,(\underset{\sim}{b_3}+\underset{\sim}{c_3})/\widetilde{a}_3\}$$

则系统(1)存在严格正的空间齐次概周期解

$$u(t)=(\hat{v}_1(t),\hat{v}_2(t),\hat{v}_3(t))$$

证明：由定理 1 的条件可得

$$0<\frac{\widetilde{c}_1+\widetilde{d}_1}{L\underset{\sim}{a_1}-\widetilde{b}_1}\leqslant 1,\ 0<\frac{\widetilde{b}_2+\widetilde{d}_2}{L\underset{\sim}{a_2}-\widetilde{c}_2}\leqslant 1,\ 0<\frac{\widetilde{b}_3+\widetilde{c}_3}{L\underset{\sim}{a_3}-\widetilde{d}_3}\leqslant 1 \tag{10}$$

记

$$m=L\max\left\{\frac{\widetilde{c}_1+\widetilde{d}_1}{L\underset{\sim}{a_1}-\widetilde{b}_1},\frac{\widetilde{b}_2+\widetilde{d}_2}{L\underset{\sim}{a_2}-\widetilde{c}_2},\frac{\widetilde{b}_3+\widetilde{c}_3}{L\underset{\sim}{a_3}-\widetilde{d}_3}\right\} \tag{11}$$

则有 $0<m\leqslant L$，且

$$(\widetilde{c}_1+\widetilde{d}_1)\frac{L}{m}\leqslant L\underset{\sim}{a_1}-\widetilde{b}_1,\ (\widetilde{b}_2+\widetilde{d}_2)\frac{L}{m}\leqslant L\underset{\sim}{a_2}-\widetilde{c}_2,\ (\widetilde{b}_3+\widetilde{c}_3)\frac{L}{m}\leqslant L\underset{\sim}{a_3}-\widetilde{d}_3 \tag{12}$$

故有

$$\widetilde{b}_1+(\widetilde{c}_1+\widetilde{d}_1)\frac{L}{m}-\underset{\sim}{f_1}m^2\leqslant L\underset{\sim}{a_1},\ \widetilde{c}_2+(\widetilde{b}_2+\widetilde{d}_2)\frac{L}{m}-\underset{\sim}{f_2}m^2\leqslant L\underset{\sim}{a_2},\ \widetilde{d}_3+(\widetilde{b}_3+\widetilde{c}_3)\frac{L}{m}-\underset{\sim}{f_3}m^2\leqslant L\underset{\sim}{a_3} \tag{13}$$

另外，由定理 1 的条件可得

$$\underset{\sim}{b_1}-\widetilde{f}_1L^2\geqslant 0,\ \underset{\sim}{c_2}-\widetilde{f}_2L^2\geqslant 0,\ \underset{\sim}{d_3}-\widetilde{f}_3L^2\geqslant 0$$

$$(\underset{\sim}{c_1}+\underset{\sim}{d_1})\frac{m}{L}\geqslant m\,\widetilde{a}_1,\ (\underset{\sim}{b_2}+\underset{\sim}{d_2})\frac{m}{L}\geqslant m\,\widetilde{a}_2$$

$$(\underset{\sim}{b_3}+\underset{\sim}{c_3})\frac{m}{L}\geqslant m\,\widetilde{a}_3 \tag{14}$$

从而得到

$$\underset{\sim}{b_1}+(\underset{\sim}{c_1}+\underset{\sim}{d_1})\frac{m}{L}-\widetilde{f}_1L^2\geqslant m\,\widetilde{a}_1$$

$$\underset{\sim}{c_2}+(\underset{\sim}{b_2}+\underset{\sim}{d_2})\frac{m}{L}-\widetilde{f}_2L^2\geqslant m\,\widetilde{a}_2$$

$$\underset{\sim}{d_3}+(\underset{\sim}{b_3}+\underset{\sim}{c_3})\frac{m}{L}-\widetilde{f}_3L^2\geqslant m\,\widetilde{a}_3 \tag{15}$$

由式(13)和式(15)可得

$$\widetilde{b}_1+(\widetilde{c}_1+\widetilde{d}_1)\frac{L}{m}-\underset{\sim}{f_1}m^2\leqslant L\underset{\sim}{a_1}$$

$$\underset{\sim}{b_1}+(\underset{\sim}{c_1}+\underset{\sim}{d_1})\frac{m}{L}-\widetilde{f}_1L^2\geqslant m\,\widetilde{a}_1 \tag{16}$$

$$\widetilde{c}_2+(\widetilde{b}_2+\widetilde{d}_2)\frac{L}{m}-\underset{\sim}{f_2}m^2\leqslant L\underset{\sim}{a_2}$$

$$\underset{\sim}{c_2}+(\underset{\sim}{b_2}+\underset{\sim}{d_2})\frac{m}{L}-\widetilde{f}_2L^2\geqslant m\,\widetilde{a}_2 \tag{17}$$

$$\widetilde{d}_3+(\widetilde{b}_3+\widetilde{c}_3)\frac{L}{m}-\underset{\sim}{f_3}m^2\leqslant L\underset{\sim}{a_3}$$

$$\underset{\sim}{d_3}+(\underset{\sim}{b_3}+\underset{\sim}{c_3})\frac{m}{L}-\widetilde{f}_3L^2\geqslant m\,\widetilde{a}_3 \tag{18}$$

定义函数集

$$H_L^m=\{\phi(t),\varphi(t),\gamma(t);\phi,\varphi,\gamma\ \text{为概周期的连续函数},\ 0<m\leqslant\phi,\varphi,\gamma\leqslant L\} \tag{19}$$

对于系统(1)考虑相应的方程

$$\begin{cases}\dot{v}_1=v_1(a_1(t)-b_1(t)v_1-c_1(t)v_2-d_1(t)v_3)+f_1(t)\\ \dot{v}_2=v_2(a_2(t)-b_2(t)v_1-c_2(t)v_2-d_2(t)v_3)+f_2(t)\\ \dot{v}_3=v_3(a_3(t)-b_3(t)v_1-c_3(t)v_2-d_3(t)v_3)+f_3(t)\end{cases} \tag{20}$$

令 $z_i=1/v_i$，则方程(20)可化为

$$\begin{cases}\dot{z}_1=b_1(t)-a_1(t)z_1+c_1(t)\dfrac{z_1}{z_2}+d_1(t)\dfrac{z_1}{z_3}-f_1(t)z_1^2\\ \dot{z}_2=c_2(t)-a_2(t)z_2+b_2(t)\dfrac{z_2}{z_1}+d_2(t)\dfrac{z_2}{z_3}-f_2(t)z_2^2\\ \dot{z}_3=d_3(t)-a_3(t)z_3+b_3(t)\dfrac{z_3}{z_1}+c_3(t)\dfrac{z_3}{z_2}-f_3(t)z_3^2\end{cases} \tag{21}$$

对于任意的 $(\phi(t),\varphi(t),\gamma(t))\in H_L^m$，由 $M[b_1]>0$，$M[c_2]>0$，$M[d_3]>0$ 知[12]，方程

$$\begin{cases}\dot{z}_1=b_1(t)-a_1(t)z_1+c_1(t)\dfrac{\phi(t)}{\varphi(t)}+d_1(t)\dfrac{\phi(t)}{\gamma(t)}-\\ \qquad f_1(t)\phi^2(t)\\ \dot{z}_2=c_2(t)-a_2(t)z_2+b_2(t)\dfrac{\varphi(t)}{\phi(t)}+d_2(t)\dfrac{\varphi(t)}{\gamma(t)}-\\ \qquad f_2(t)\varphi^2(t)\\ \dot{z}_3=d_3(t)-a_3(t)z_3+b_3(t)\dfrac{\gamma(t)}{\phi(t)}+c_3(t)\dfrac{\gamma(t)}{\varphi(t)}-\\ \qquad f_3(t)\gamma^2(t)\end{cases} \tag{22}$$

有一个概周期解

$$\begin{cases}\hat{z}_1(t)=\int_{-\infty}^{t}e^{-\int_s^t a_1(r)\mathrm{d}r}\Big[b_1(s)+c_1(s)\dfrac{\phi(s)}{\varphi(s)}+\\ \qquad d_1(s)\dfrac{\phi(s)}{\gamma(s)}-f_1(s)\phi^2(s)\Big]\mathrm{d}s\\ \hat{z}_2(t)=\int_{-\infty}^{t}e^{-\int_s^t a_2(r)\mathrm{d}r}\Big[c_2(s)+b_2(s)\dfrac{\varphi(s)}{\phi(s)}+\\ \qquad d_2(s)\dfrac{\varphi(s)}{\gamma(s)}-f_2(s)\varphi^2(s)\Big]\mathrm{d}s\\ \hat{z}_3(t)=\int_{-\infty}^{t}e^{-\int_s^t a_2(r)\mathrm{d}r}\Big[d_3(s)+b_3(s)\dfrac{\gamma(s)}{\phi(s)}+\\ \qquad c_3(s)\dfrac{\gamma(s)}{\varphi(s)}-f_3(s)\gamma^2(s)\Big]\mathrm{d}s\end{cases} \tag{23}$$

现在利用式(23)定义如下映射 A:

$$A(\phi,\varphi,\gamma)=(\hat{z}_1,\hat{z}_2,\hat{z}_3),\ \forall(\phi,\varphi,\gamma)\in H_L^m \tag{24}$$

由式(16)~(18)及式(23)知

$$\hat{z}_1(t)\geq\int_{-\infty}^{t}e^{-\tilde{a}_1(t-s)}\Big[\underset{\sim}{b}_1+(\underset{\sim}{c}_1+\underset{\sim}{d}_1)\frac{m}{L}-\tilde{f}_1L^2\Big]\mathrm{d}s=\frac{1}{\tilde{a}_1}\Big[\underset{\sim}{b}_1+(\underset{\sim}{c}_1+\underset{\sim}{d}_1)\frac{m}{L}-\tilde{f}_1L^2\Big]\geq m \tag{25}$$

$$\hat{z}_1(t)\leq\int_{-\infty}^{t}e^{-\underset{\sim}{a}_1(t-s)}\Big[\tilde{b}_1+(\tilde{c}_1+\tilde{d}_1)\frac{L}{m}-\underset{\sim}{f}_1m^2\Big]\mathrm{d}s=\frac{1}{\underset{\sim}{a}_1}\Big[\tilde{b}_1+(\tilde{c}_1+\tilde{d}_1)\frac{L}{m}-\underset{\sim}{f}_1m^2\Big]\leq L \tag{26}$$

$$\hat{z}_2(t)\geq\int_{-\infty}^{t}e^{-\tilde{a}_2(t-s)}\Big[\underset{\sim}{c}_2+(\underset{\sim}{b}_2+\underset{\sim}{d}_2)\frac{m}{L}-\tilde{f}_2L^2\Big]\mathrm{d}s=\frac{1}{\tilde{a}_2}\Big[\underset{\sim}{c}_2+(\underset{\sim}{b}_2+\underset{\sim}{d}_2)\frac{m}{L}-\tilde{f}_2L^2\Big]\geq m \tag{27}$$

$$\hat{z}_2(t)\leq\int_{-\infty}^{t}e^{-\underset{\sim}{a}_2(t-s)}\Big[\tilde{c}_2+(\tilde{b}_2+\tilde{d}_2)\frac{L}{m}-\underset{\sim}{f}_2m^2\Big]\mathrm{d}s=\frac{1}{\underset{\sim}{a}_2}\Big[\tilde{c}_2+(\tilde{b}_2+\tilde{d}_2)\frac{L}{m}-\underset{\sim}{f}_2m^2\Big]\leq L \tag{28}$$

$$\hat{z}_3(t)\geq\int_{-\infty}^{t}e^{-\tilde{a}_3(t-s)}\Big[\underset{\sim}{d}_3+(\underset{\sim}{b}_3+\underset{\sim}{c}_3)\frac{m}{L}-\tilde{f}_3L^2\Big]\mathrm{d}s=\frac{1}{\tilde{a}_3}\Big[\underset{\sim}{d}_3+(\underset{\sim}{b}_3+\underset{\sim}{c}_3)\frac{m}{L}-\tilde{f}_3L^2\Big]\geq m \tag{29}$$

$$\hat{z}_3(t)\leq\int_{-\infty}^{t}e^{-\underset{\sim}{a}_3(t-s)}\Big[\tilde{d}_3+(\tilde{b}_3+\tilde{c}_3)\frac{L}{m}-\underset{\sim}{f}_3m^2\Big]\mathrm{d}s=\frac{1}{\underset{\sim}{a}_3}\Big[\tilde{d}_3+(\tilde{b}_3+\tilde{c}_3)\frac{L}{m}-\underset{\sim}{f}_3m^2\Big]\leq L \tag{30}$$

从而$(\hat{z}_1,\hat{z}_2,\hat{z}_3)\in H_L^m$,即 $AH_L^m\subset H_L^m$,若 A 还是一致有界和等度连续的,则由 Ascoli-Arzela 定理[11]知 A 是紧映射.

一致有界性显然. 事实上,对于任意的$(\phi,\varphi,\gamma)\in H_L^m$,有$(\hat{z}_1,\hat{z}_2,\hat{z}_3)=A(\phi,\varphi,\gamma)\in H_L^m$,即满足 $0<(m,m,m)\leq A(\phi,\varphi,\gamma)=(\hat{z}_1,\hat{z}_2,\hat{z}_3)\leq(L,L,L)$

下面证明等度连续性. 对任意的$(\phi,\varphi,\gamma)\in H_L^m$,记$(\hat{z}_1,\hat{z}_2,\hat{z}_3)=A(\phi,\varphi,\gamma)$,$t_1<t_2$,则有

$$|\hat{z}_1(t_1)-\hat{z}_1(t_2)|=\Big|\int_{-\infty}^{t_1}e^{-\int_s^{t_1}a_1(r)\mathrm{d}r}\Big[b_1(s)+c_1(s)\frac{\phi(s)}{\varphi(s)}+d_1(s)\frac{\phi(s)}{\gamma(s)}-f_1(s)\phi^2(s)\Big]\mathrm{d}s-\int_{-\infty}^{t_2}e^{-\int_s^{t_2}a_1(r)\mathrm{d}r}\Big[b_1(s)+c_1(s)\frac{\phi(s)}{\varphi(s)}+d_1(s)\frac{\phi(s)}{\gamma(s)}-f_1(s)\phi^2(s)\Big]\mathrm{d}s\Big| \tag{31}$$

再记

$$h_1(t)=b_1(t)+c_1(t)\frac{\phi(t)}{\varphi(t)}+d_1(t)\frac{\phi(t)}{\gamma(t)}-f_1(t)\phi^2(t)$$

则有

$$\begin{aligned}|\hat{z}_1(t_2)-\hat{z}_1(t_1)|&=\Big|\int_{-\infty}^{t_2}e^{-\int_s^{t_2}a_1(r)\mathrm{d}r}h_1(s)\mathrm{d}s-\int_{-\infty}^{t_1}e^{-\int_s^{t_1}a_1(r)\mathrm{d}r}h_1(s)\mathrm{d}s\Big|\leq\\ &\Big|\int_{t_1}^{t_2}e^{-\int_s^{t_2}a_1(r)\mathrm{d}r}h_1(s)\mathrm{d}s\Big|+\\ &\Big|\int_{-\infty}^{t_1}e^{-\int_s^{t_1}a_1(r)\mathrm{d}r}\big(e^{-\int_{t_1}^{t_2}a_1(r)\mathrm{d}r}-1\big)h_1(s)\mathrm{d}s\Big|\end{aligned} \tag{32}$$

由于$(\phi,\varphi,\gamma)\in H_L^m$,则存在正常数 M 使得$|h_1(s)|\leq M$,从而式(32)可化为

$$|\hat{z}_1(t_2)-\hat{z}_1(t_1)|\leq Me^{-\int_{\xi_1}^{t_2}a_1(r)\mathrm{d}r}|t_2-t_1|+\frac{1}{\underset{\sim}{a}_1}M\Big|1-e^{-\int_{t_1}^{t_2}a_1(r)\mathrm{d}r}\Big| \tag{33}$$

式中 ξ_1 位于 t_1 与 t_2 之间. 类似地,可得

$$|\hat{z}_2(t_2)-\hat{z}_2(t_1)|\leq Ne^{-\int_{\xi_2}^{t_2}a_2(r)\mathrm{d}r}|t_1-t_2|+\frac{1}{\underset{\sim}{a}_2}N\Big|1-e^{-\int_{t_1}^{t_2}a_2(r)\mathrm{d}r}\Big| \tag{34}$$

式中 N 为正常数.

同样可得

$$|\hat{z}_3(t_2)-\hat{z}_3(t_1)|\leq Pe^{-\int_{\xi_3}^{t_2}a_3(r)\mathrm{d}r}|t_1-t_2|+\frac{1}{\underset{\sim}{a}_3}P\Big|1-e^{-\int_{t_1}^{t_2}a_3(r)\mathrm{d}r}\Big| \tag{35}$$

式中 P 为正常数.

由式(33)～式(35)知，对任意$(\phi, \varphi, \gamma) \in H_L^m$，则

$$\lim_{\xi\to 0}\sup_{|t_1-t_2|\leqslant\xi}|A(\phi,\varphi,\gamma)(t_1)-A(\phi,\varphi,\gamma)(t_2)|=0$$

一致成立. 故由式(24)所定义的映射 A 是 H_L^m 到其自身的紧映射，由引理2知，A 在 H_L^m 中存在不动点(ϕ,φ,γ)，它就是式(21)的解，从而式(20)有严格正的概周期解$(v_1^*(t), v_2^*(t), v_3^*(t)) = (1/\phi(t), 1/\varphi(t), 1/\gamma(t))$，$t\in\mathbb{R}^+$. 显然$(v_1^*(t), v_2^*(t), v_3^*(t))$，$t\in\mathbb{R}^+$也是满足方程(1)的空间齐次概周期解.

定理 2　若系统(1)满足下述条件：

(ⅰ) 定理1的条件；

(ⅱ)

$$\begin{cases}\sup\limits_{t\geqslant 0}(b_3(t)+b_2(t)-b_1(t)) = -\varepsilon_1<0\\ \sup\limits_{t\geqslant 0}(c_3(t)+c_1(t)-c_2(t)) = -\varepsilon_2<0\\ \sup\limits_{t\geqslant 0}(d_1(t)+d_2(t)-d_3(t)) = -\varepsilon_3<0\end{cases}$$

则系统(1)存在严格正的空间齐次概周期解$(v_1^*(t), v_2^*(t), v_3^*(t))$，并且还是全局稳定的，即系统(1)～(3)的解$(v_1(x,t), v_2(x,t), v_3(x,t))$，$(x,t)\in\overline{\Omega}\times\mathbb{R}^+$均满足：

$$\lim_{t\to\infty}(v_i(x,t)-v_i^*(t))=0,\ i=1,2,3,\ x\in\overline{\Omega} \quad (36)$$

证明：由定理1知其存在性. 对于稳定性，对式(36)就初值 $v_{i0}(x)$ $(i=1,2,3)$ 分2种情况给予证明.

1) $v_{i0}(x)>0, x\in\overline{\Omega}$;

2) 存在 $x_0\in\overline{\Omega}$，使得 $v_{10}(x_0)=0, v_{20}(x_0)=0$ 或 $v_{30}(x_0)=0$.

对情形1)，记 $l_i=\min\limits_{\overline{\Omega}} v_{i0}(x)$，$r_i=\max\limits_{\overline{\Omega}} v_{i0}(x)$，则 $0<l_i\leqslant v_{i0}(x)\leqslant r_i$. 设$(\bar v_1(t), \bar v_2(t), \bar v_3(t))$，$(\underline{v}_1(t), \underline{v}_2(t), \underline{v}_3(t))$是方程(20)分别过初值$(\bar v_1(0), \bar v_2(0), \bar v_3(0))=(r_1,r_2,r_3)$，$(\underline{v}_1(0), \underline{v}_2(0), \underline{v}_3(0))=(l_1,l_2,l_3)$的解，则$(\bar v_1(t), \bar v_2(t), \bar v_3(t))$，$(\underline{v}_1(t), \underline{v}_2(t), \underline{v}_3(t))$为系统(1)～(3)的一对有序上、下解，由引理1知系统(1)～(3)存在唯一解：

$$(v_1(x,t), v_2(x,t), v_3(x,t)),\ (x,t)\in\overline{\Omega}\times\mathbb{R}^+$$

并满足

$$(\underline{v}_1(t), \underline{v}_2(t), \underline{v}_3(t)) \leqslant (v_1(x,t), v_2(x,t), v_3(x,t)) \leqslant (\bar v_1(t), \bar v_2(t), \bar v_3(t)) \quad (37)$$

若有

$$\lim_{t\to\infty}\bar v_i(t)-v_i^*(t)=\lim_{t\to\infty}\underline{v}_i(t)-v_i^*(t)=0 \quad (38)$$

则必有式(36)成立，故要证式(38)成立只须证对任意的正初值$(v_1(0), v_2(0), v_3(0))=(v_{10}, v_{20}, v_{30})$，方程(20)的相应解$(v_1(t), v_2(t), v_3(t))$满足

$$\lim_{t\to\infty}(v_i(t)-v_i^*(t))=0, i=1,2,3 \quad (39)$$

因初值$(v_{10}, v_{20}, v_{30})>0$，放牧率$(\underset{\sim}{f_1}, \underset{\sim}{f_2}, \underset{\sim}{f_3})>0$，由生物学意义知$(v_1(t), v_2(t), v_3(t))>0$. 记

$$P_i(t)=\ln v_i(t), Q_i(t)=\ln v_i^*(t), i=1,2,3 \quad (40)$$

$$\begin{cases}\dfrac{\mathrm{d}}{\mathrm{d}t}(P_1(t)-Q_1(t)) = -b_1(t)(\mathrm{e}^{P_1(t)}-\mathrm{e}^{Q_1(t)}) - c_1(t)(\mathrm{e}^{P_2(t)}-\mathrm{e}^{Q_2(t)}) - d_1(t)(\mathrm{e}^{P_3(t)}-\mathrm{e}^{Q_3(t)}) + \left(\dfrac{1}{v_1(t)}-\dfrac{1}{v_1^*(t)}\right)f_1(t)\\ \dfrac{\mathrm{d}}{\mathrm{d}t}(P_2(t)-Q_2(t)) = -b_2(t)(\mathrm{e}^{P_1(t)}-\mathrm{e}^{Q_1(t)}) - c_2(t)(\mathrm{e}^{P_2(t)}-\mathrm{e}^{Q_2(t)}) - d_2(t)(\mathrm{e}^{P_3(t)}-\mathrm{e}^{Q_3(t)}) + \left(\dfrac{1}{v_2(t)}-\dfrac{1}{v_2^*(t)}\right)f_2(t)\\ \dfrac{\mathrm{d}}{\mathrm{d}t}(P_3(t)-Q_3(t)) = -b_3(t)(\mathrm{e}^{P_1(t)}-\mathrm{e}^{Q_1(t)}) - c_3(t)(\mathrm{e}^{P_2(t)}-\mathrm{e}^{Q_2(t)}) - d_3(t)(\mathrm{e}^{P_3(t)}-\mathrm{e}^{Q_3(t)}) + \left(\dfrac{1}{v_3(t)}-\dfrac{1}{v_3^*(t)}\right)f_3(t)\end{cases} \quad (41)$$

即

$$\begin{cases}\dfrac{\mathrm{d}}{\mathrm{d}t}(P_1(t)-Q_1(t)) = -\left(b_1(t)+\dfrac{f_1(t)}{v_1(t)v_1^*(t)}\right)\cdot(\mathrm{e}^{P_1(t)}-\mathrm{e}^{Q_1(t)}) - c_1(t)(\mathrm{e}^{P_2(t)}-\mathrm{e}^{Q_2(t)}) - d_1(t)(\mathrm{e}^{P_3(t)}-\mathrm{e}^{Q_3(t)})\\ \dfrac{\mathrm{d}}{\mathrm{d}t}(P_2(t)-Q_2(t)) = -b_2(t)(\mathrm{e}^{P_1(t)}-\mathrm{e}^{Q_1(t)}) - \left(c_2(t)+\dfrac{f_2(t)}{v_2(t)v_2^*(t)}\right)(\mathrm{e}^{P_2(t)}-\mathrm{e}^{Q_2(t)}) - d_2(t)(\mathrm{e}^{P_3(t)}-\mathrm{e}^{Q_3(t)})\\ \dfrac{\mathrm{d}}{\mathrm{d}t}(P_3(t)-Q_3(t)) = -b_3(t)(\mathrm{e}^{P_1(t)}-\mathrm{e}^{Q_1(t)}) - c_3(t)(\mathrm{e}^{P_2(t)}-\mathrm{e}^{Q_2(t)}) - \left(d_3(t)+\dfrac{f_3(t)}{v_3(t)v_3^*(t)}\right)(\mathrm{e}^{P_3(t)}-\mathrm{e}^{Q_3(t)})\end{cases} \quad (42)$$

考虑 Lyapunov 函数

$$U(t)=\sum_{i=1}^{3}|P_i(t)-Q_i(t)|, t\geqslant 0$$

并记 D^+U 为 U 的右上导数，则有

$$D^+U(t)=\sum_{i=1}^{3}D^+|P_i(t)-Q_i(t)|=\sum_{i=1}^{3}\operatorname{sgn}(P_i(t)-Q_i(t))\frac{\mathrm{d}}{\mathrm{d}t}(P_i(t)-Q_i(t))=\operatorname{sgn}(P_1(t)-Q_1(t))\Big[-\Big(b_1(t)+\frac{f_1(t)}{v_1(t)v_1^*(t)}\Big)\cdot(\mathrm{e}^{P_1(t)}-\mathrm{e}^{Q_1(t)})-c_1(t)(\mathrm{e}^{P_2(t)}-\mathrm{e}^{Q_2(t)})-d_1(t)(\mathrm{e}^{P_3(t)}-\mathrm{e}^{Q_3(t)})\Big]+\operatorname{sgn}(P_2(t)-Q_2(t))\Big[-b_2(t)(\mathrm{e}^{P_1(t)}-\mathrm{e}^{Q_1(t)})-\Big(c_2(t)+\frac{f_2(t)}{v_2(t)v_2^*(t)}\Big)(\mathrm{e}^{P_2(t)}-\mathrm{e}^{Q_2(t)})-d_2(t)(\mathrm{e}^{P_3(t)}-\mathrm{e}^{Q_3(t)})\Big]+\operatorname{sgn}(P_3(t)-Q_3(t))\Big[-b_3(t)(\mathrm{e}^{P_1(t)}-\mathrm{e}^{Q_1(t)})-c_3(t)(\mathrm{e}^{P_2(t)}-\mathrm{e}^{Q_2(t)})-\Big(d_3(t)+\frac{f_3(t)}{v_3(t)v_3^*(t)}\Big)(\mathrm{e}^{P_3(t)}-\mathrm{e}^{Q_3(t)})\Big]\leqslant(b_3(t)+b_2(t)-b_1(t))|\mathrm{e}^{P_1(t)}-\mathrm{e}^{Q_1(t)}|+(c_1(t)+c_3(t)-c_2(t))|\mathrm{e}^{P_2(t)}-\mathrm{e}^{Q_2(t)}|+(d_1(t)+d_2(t)-d_3(t))|\mathrm{e}^{P_3(t)}-\mathrm{e}^{Q_3(t)}|\leqslant-\varepsilon_1|v_1(t)-v_1^*(t)|-\varepsilon_2|v_2(t)-v_2^*(t)|-\varepsilon_3|v_3(t)-v_3^*(t)| \tag{43}$$

积分得

$$U(t)+\sum_{i=1}^{3}\varepsilon_i\int_0^t|v_i(s)-v_i^*(s)|\mathrm{d}s\leqslant U(0) \tag{44}$$

由 $U(t)$ 的非负性及 $U(0)$ 的有界性知 $U(t)$ 是有界的，且 $\int_0^t|v_i(t)-v_i^*(t)|\mathrm{d}s, i=1,2,3$ 收敛，由式(43)还可知 $D^+U(t)<0$，则极限

$$\lim_{t\to\infty}U(t)=l \tag{45}$$

存在且 $U(t)\geqslant l$. 若 $l>0$，则

$$|P_1(t)-Q_1(t)|>\frac{l}{4},|P_2(t)-Q_2(t)|>\frac{l}{4}$$

$$|P_3(t)-Q_3(t)|>\frac{l}{4}$$

至少有一个成立，不妨设 $|P_1(t)-Q_1(t)|>\frac{l}{4}$，从而 $P_1(t)$ 与 $Q_1(t)$ 无交点. 设 $P_1(t)>Q_1(t)$，即有

$$P_1(t)-Q_1(t)>\frac{l}{4}$$

故

$$\int_0^t|v_1(t)-v_1^*(t)|\mathrm{d}s=\int_0^t|\mathrm{e}^{P_1(s)}-\mathrm{e}^{Q_1(s)}|\mathrm{d}s=\int_0^t\mathrm{e}^{Q_1(s)}|\mathrm{e}^{P_1(s)-Q_1(s)}-1|\mathrm{d}s\geqslant m\int_0^t(\mathrm{e}^{P_1(s)-Q_1(s)}-1)\mathrm{d}s>m\int_0^t(\mathrm{e}^{l/4}-1)\mathrm{d}s=m(\mathrm{e}^{l/4}-1)t\to+\infty$$

这与 $\int_0^t|v_i(s)-v_i^*(s)|\mathrm{d}s$ 收敛矛盾. 故 $l=0$，从而

$$\lim_{t\to\infty}|v_i(t)-v_i^*(t)|=0,i=1,2,3 \tag{46}$$

即式(39)成立.

对于情形 2)，先选择充分大的正数 M_1、M_2、M_3，使

$$\begin{cases}f_1(t)\leqslant-M_1(a_1(t)-b_1(t)M_1),\ t>0\\ f_2(t)\leqslant-M_2(a_2(t)-c_2(t)M_2),\ t>0\\ f_3(t)\leqslant-M_3(a_3(t)-d_3(t)M_3),\ t>0\end{cases} \tag{47}$$

且 $M_i\geqslant\max_{x\in\bar{\Omega}}v_{i0}(x), i=1,2,3$. 记 $\underset{\sim}{v_i}=0,\tilde{v}_i=M_i$，则有

$$\begin{cases}\dfrac{\partial\tilde{v}_1}{\partial t}-k_1(t)\Delta\tilde{v}_1-\tilde{v}_1[a_1(t)-b_1(t)\tilde{v}_1-c_1(t)\underset{\sim}{v_2}-d_1(t)\underset{\sim}{v_3}]-f_1(t)\geqslant0\\ \dfrac{\partial\underset{\sim}{v_1}}{\partial t}-k_1(t)\Delta\underset{\sim}{v_1}-\underset{\sim}{v_1}[a_1(t)-b_1(t)\underset{\sim}{v_1}-c_1(t)\tilde{v}_2-d_1(t)\tilde{v}_3]-f_1(t)\leqslant0\end{cases} \tag{48}$$

$$\begin{cases}\dfrac{\partial\tilde{v}_2}{\partial t}-k_2(t)\Delta\tilde{v}_2-\tilde{v}_2[a_2(t)-b_2(t)\underset{\sim}{v_1}-c_2(t)\tilde{v}_2-d_2(t)\underset{\sim}{v_3}]-f_2(t)\geqslant0\\ \dfrac{\partial\underset{\sim}{v_2}}{\partial t}-k_2(t)\Delta\underset{\sim}{v_2}-\underset{\sim}{v_2}[a_2(t)-b_2(t)\tilde{v}_1-c_2(t)\underset{\sim}{v_2}-d_2(t)\tilde{v}_3]-f_2(t)\leqslant0\end{cases} \tag{49}$$

$$\begin{cases}\dfrac{\partial\tilde{v}_3}{\partial t}-k_3(t)\Delta\tilde{v}_3-\tilde{v}_3[a_3(t)-b_3(t)\underset{\sim}{v_1}-c_3(t)\underset{\sim}{v_2}-d_3(t)\tilde{v}_3]-f_3(t)\geqslant0\\ \dfrac{\partial\underset{\sim}{v_3}}{\partial t}-k_3(t)\Delta\underset{\sim}{v_3}-\underset{\sim}{v_3}[a_3(t)-b_3(t)\tilde{v}_1-c_3(t)\tilde{v}_2-d_3(t)\underset{\sim}{v_3}]-f_3(t)\leqslant0\end{cases} \tag{50}$$

即 $\underset{\sim}{v_i}=0,\tilde{v}_i=M_i,i=1,2,3$，是系统 (1) ~ (3) 的一对有序上、下解，由引理 1 知，系统 (1) ~ (3) 存在唯一解 $(v_1(x,t),v_2(x,t),v_3(x,t))$ 并满足

$$0\leqslant v_i(x,t)\leqslant M_i,\ i=1,2,3,\ (x,t)\in\bar{\Omega}\times[0,\infty) \tag{51}$$

再选择正常数 δ_1、δ_2、δ_3，使得

$$\begin{cases}\delta_1 + a_1(t) - b_1(t)v_1(x,t) - c_1(t)v_2(x,t) - \\ \quad d_1(t)v_3(x,t) > 0 \\ \delta_2 + a_2(t) - b_2(t)v_1(x,t) - c_2(t)v_2(x,t) - \\ \quad d_2(t)v_3(x,t) > 0 \\ \delta_3 + a_3(t) - b_3(t)v_1(x,t) - c_3(t)v_2(x,t) - \\ \quad d_3(t)v_3(x,t) > 0\end{cases} \tag{52}$$

从而

$$\begin{cases}\frac{\partial v_1}{\partial t} - k_1(t)\Delta v_1 + \delta_1 v_1 = v_1[\delta_1 + a_1(t) - b_1(t)v_1 - \\ \quad c_1(t)v_2 - d_1(t)v_3] + f_1(t) \geqslant 0 \\ \frac{\partial v_2}{\partial t} - k_2(t)\Delta v_2 + \delta_2 v_2 = v_2[\delta_2 + a_2 - b_2(t)v_1 - \\ \quad c_2(t)v_2 - d_2(t)v_3] + f_2(t) \geqslant 0 \\ \frac{\partial v_3}{\partial t} - k_3(t)\Delta v_3 + \delta_3 v_3 = v_3[\delta_3 + a_3 - b_3(t)v_1 - \\ \quad c_3(t)v_2 - d_3(t)v_3] + f_3(t) \geqslant 0\end{cases} \tag{53}$$

证明在 $\overline{\Omega}\times(0,\infty)$ 上有 $v_i(x,t)>0, i=1,2,3$，先证在 $\Omega\times(0,\infty)$ 上有 $v_i(x,t)>0$. 若存在 $(x_0,t_0)\in\Omega\times(0,\infty)$ 使得 $v_i(x_0,t_0)=0$，则由极值原理知，在 $\overline{\Omega}\times[0,t_0]$ 上有 $v_i(x,t)\equiv 0$. 但初值 $v_i(x,0)=v_{i0}(x)\geqslant 0$，且不恒为 0，矛盾. 故在 $\Omega\times(0,\infty)$ 上有 $v_i(x,t)>0$. 再证在 $\partial\Omega\times(0,\infty)$ 上有 $v_i(x,t)>0$. 若存在 $(x_0,t_0)\in\partial\Omega\times(0,\infty)$ 使得 $v_i(x_0,t_0)=0$，则由边界形式的极值原理知 $\frac{\partial v_i(x,t)}{\partial n}<0$，$(x,t)\in\partial\Omega\times(0,\infty)$，这又与边界条件（2）矛盾. 所以在 $\overline{\Omega}\times(0,\infty)$ 上，恒有 $v_i(x,t)>0$. 对于固定的 $\lambda>0$，由式(51)有

$$0 < v_i(x,\lambda) \leqslant M_i, i=1,2,3,\ x\in\overline{\Omega} \tag{54}$$

由于 $v_i(x,t+\lambda)$ 在 $\overline{\Omega}\times(0,\infty)$ 上满足式(1)，在 $\partial\Omega\times(0,\infty)$ 上满足式(2)，故 $(v_1(x,t+\lambda), v_2(x,t+\lambda), v_3(x,t+\lambda))$ 可视为过初值 $(\hat{v}_{10}(x),\hat{v}_{20}(x),\hat{v}_{30}(x))=(v_1(x,\lambda),v_2(x,\lambda),v_3(x,\lambda))$ 的解，而在 $\overline{\Omega}$ 上有 $\hat{v}_{i0}>0$，$i=1,2,3$，再由情形 1）的结论有

$$\lim_{t\to\infty}(v_i(x,t+\lambda) - v_i^*(t)) = 0, i=1,2,3$$

关于 $x\in\overline{\Omega}$ 一致成立. 由 λ 的任意性知

$$\lim_{t\to\infty}(v_i(x,t) - v_i^*(t)) = 0, i=1,2,3$$

关于 $x\in\overline{\Omega}$ 一致成立.

证毕

4 结论

1）构造了所研究模型的上、下解及 Lyapunov 函数.

2）获得了所研究模型的正的空间齐次概周期解的存在性的充分条件.

3）证明了所研究模型的任意解关于其正的空间齐次概周期解是全局渐近稳定的.

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